# Sketching Techniques for Massive Data Streams

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## Disclaimers



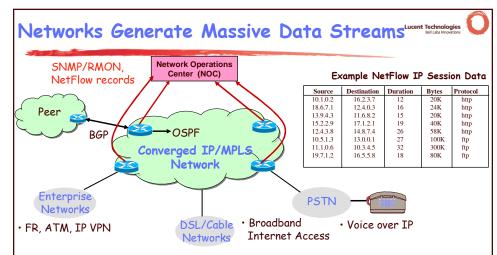
- Personal, biased view of data-streaming world
  - Revolve around own line of work, interests, and results
  - Focus on a couple of basic algorithmic tools
    - A lot more out there . . .
    - Interesting research prototypes and systems work not covered Aurora, STREAM, Telegraph, . . .
- Discussion necessarily short and fairly high-level
  - More detailed overviews
    - 3-hour tutorial at VLDB'02, Motwani et al. [PODS'02], overview article by S. Muthukrishnan
  - Ask questions!
  - Talk to me afterwards

#### Data-Stream Management

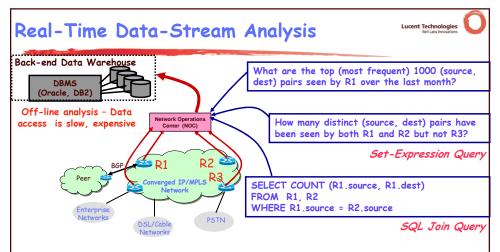


- Traditional DBMS data stored in finite, persistent data sets
- Data Streams distributed, continuous, unbounded, rapid, time varying, noisy, . . .
- Data-Stream Management variety of modern applications
  - Network monitoring and traffic engineering
  - Telecom call-detail records
  - Network security
  - Financial applications
  - Sensor networks
  - Manufacturing processes
  - Web logs and clickstreams
  - Massive data sets

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- SNMP/RMON/NetFlow data records arrive 24x7 from different parts of the network
- Truly massive streams arriving at rapid rates
  - AT&T collects 600-800 GigaBytes of NetFlow data each day!
- Typically shipped to a back-end data warehouse (off site) for off-line analysis

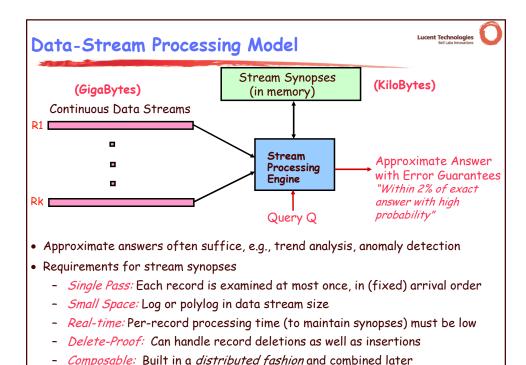


- Need ability to process/analyze network-data streams in real-time
  - As records stream in: look at records only once in arrival order!
  - Within resource (CPU, memory) limitations of the NOC
- Critical to important NM tasks
  - Detect and react to Fraud, Denial-of-Service attacks, SLA violations
  - Real-time traffic engineering to improve load-balancing and utilization

Talk Outline



- Introduction & Motivation
- Data Stream Computation Model
- Two Basic Sketching Tools for Streams
  - -Linear-Projection (aka AMS) Sketches
    - · Applications: Join/Multi-Join Queries, Wavelets
  - -Hash (aka FM) Sketches
    - · Applications: Distinct Values, Set Expressions
- Extensions
  - -Correlating XML data streams
- Conclusions & Future Research Directions



# Synopses for Relational Streams





- Quantiles and 1-d histograms [MRL98,99], [GK01], [GKM502]
  - · Cannot capture attribute correlations
  - · Little support for approximation guarantees
  - Samples (e.g., using Reservoir Sampling)
    - Perform poorly for joins [AGMS99] or distinct values [CCMN00]
    - · Cannot handle deletion of records
  - Multi-d histograms/wavelets
    - · Construction requires multiple passes over the data
- Different approach: Pseudo-random sketch synopses
  - Only logarithmic space
  - Probabilistic quarantees on the quality of the approximate answer
  - Support insertion as well as deletion of records

# Linear-Projection (aka AMS) Sketch Synopses Multiple Control of the Control of th



 Goal: Build small-space summary for distribution vector f(i) (i=1,..., N) seen as a stream of i-values

Data stream: 3, 1, 2, 4, 2, 3, 5, ...

 <u>Basic Construct:</u> Randomized Linear Projection of f() = project onto inner/dot product of f-vector

 $< f, \xi> = \sum f(i) \xi_i \quad \text{ where } \xi \text{ = vector of random values from an appropriate distribution}$ 

– Simple to compute over the stream: Add  $\xi_i$  whenever the i-th value is seen

Data stream: 3, 1, 2, 4, 2, 3, 5, ...  $\xi_1 + 2\xi_2 + 2\xi_3 + \xi_4 + \xi_5$ 

- Generate  $oldsymbol{\xi}_i$  's in small (logN) space using pseudo-random generators
- Tunable probabilistic guarantees on approximation error
- *Delete-Proof:* Just subtract  $\xi_i$  to delete an i-th value occurrence
- Composable: Simply add independently-built projections

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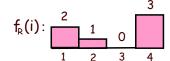
# Example: Binary-Join COUNT Query



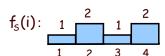


• Example:

Data stream R.A: 4 1 2 4 1 4



Data stream S.A: 3 1 2 4 2 4



COUNT(R 
$$\bowtie_A S$$
) =  $\sum_i f_R(i) \cdot f_S(i)$   
= 10 (2 + 2 + 0 + 6)

- Exact solution: too expensive, requires O(N) space!
  - N = sizeof(domain(A))

## Basic AMS Sketching Technique [AMS96]



- Key Intuition: Use randomized linear projections of f() to define random variable X such that
  - X is easily computed over the stream (in small space)
  - $-E[X] = COUNT(R \bowtie_A S)$
  - Var[X] is small



Probabilistic error guarantees (e.g., actual answer is 10±1 with probability 0.9)

- Basic Idea:
  - Define a family of 4-wise independent {-1, +1} random variables

$$\{\xi_i : i = 1,...,N\}$$

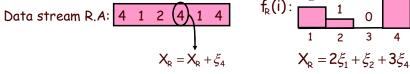
- $Pr[\xi_i = +1] = Pr[\xi_i = -1] = 1/2$ 
  - Expected value of each  $\xi_i$ ,  $E[\xi_i] = 0$
- Variables  $\xi_i$  are 4-wise independent
  - Expected value of product of 4 distinct  $\xi_i = 0$
- Variables  $\xi_i$  can be generated using pseudo-random generator using only O(log N) space (for seeding)!

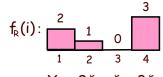
## AMS Sketch Construction





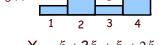
- Compute random variables:  $X_R = \sum_i f_R(i) \xi_i$  and  $X_S = \sum_i f_S(i) \xi_i$ 
  - Simply add  $\xi_i$  to  $X_R(X_S)$  whenever the i-th value is observed in the R.A (S.A) stream
- Define  $X = X_R X_S$  to be estimate of COUNT query
- Example:

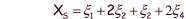




Data stream 5.A: 3(1) 2 4 2 4  $f_5(i)$ : 1 2 3 4







# Binary-Join AMS Sketching Analysis



• Expected value of X = COUNT(R  $\bowtie_A S$ )

$$\begin{split} E[X] &= E[X_R \cdot X_S] \\ &= E[\sum_i f_R(i) \xi_i \cdot \sum_i f_S(i) \xi_i] \\ &= E[\sum_i f_R(i) \cdot f_S(i) \xi_i^2] + E[\sum_{i \neq i'} f_R(i) \cdot f_S(i') \xi_i \xi_{i'}] \\ &= \sum_i f_R(i) \cdot f_S(i) \end{split}$$

• Using 4-wise independence, possible to show that

$$Var[X] \le 2 \cdot SJ(R) \cdot SJ(S)$$

•  $SJ(R) = \sum_{i} f_{R}(i)^{2}$  is <u>self-join size of R</u>

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# **Boosting Accuracy**



• Chebyshev's Inequality:

$$\Pr(|X-E[X]| \ge \epsilon E[X]) \le \frac{Var[X]}{\epsilon^2 E[X]^2}$$

 $\bullet$  Boost accuracy to  $\epsilon$  by averaging over several independent copies of X (reduces variance)

$$S = \frac{8 \cdot (2 \cdot SJ(R) \cdot SJ(S))}{\epsilon^2 COUNT^2} \quad \text{copies} \quad E[Y] = E[X] = COUNT(R \bowtie S)$$

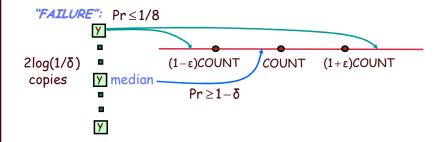
• By Chebyshev:  $Var[Y] = \frac{Var[X]}{s} \le \frac{\epsilon^2 COUNT^2}{8}$   $Pr(|Y - COUNT| \ge \epsilon \cdot COUNT) \le \frac{Var[Y]}{\epsilon^2 COUNT^2} \le \frac{1}{8}$ 

# Boosting Confidence





- $\bullet$  Boost confidence to  $1-\delta$  by taking median of  $2log(1/\delta$  ) independent copies of Y
- Each Y = Binomial Trial



 $Pr[|median(Y)-COUNT| \ge \varepsilon \cdot COUNT]$ 

=  $Pr[\# failures in 2log(1/\delta) trials >= log(1/\delta)]$ 

 $\leq \delta$  (By Chernoff Bound)

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## Summary of Binary-Join AMS Sketching





- Step 1: Compute random variables:  $X_R = \sum_i f_R(i) \xi_i$  and  $X_S = \sum_i f_S(i) \xi_i$
- Step 2: Define X= X<sub>R</sub>X<sub>S</sub>
- Steps 3 & 4: Average independent copies of X; Return median of averages

• <u>Main Theorem (AGMS99)</u>: Sketching approximates COUNT to within a relative error of  $\epsilon$  with probability  $\geq 1-\delta$  using space

$$O(\frac{SJ(R) \cdot SJ(S) \cdot log(1/\delta) \cdot logN}{\epsilon^2 COUNT^2})$$

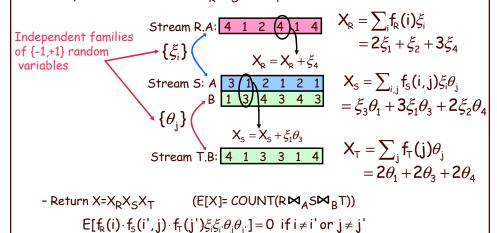
- Remember: O(log N) space for "seeding" the construction of each X

#### AMS Sketching for Multi-Join Aggregates Lucent Technologies [DGGR02]





- <u>Problem</u>: Compute answer for COUNT( $R\bowtie_A S\bowtie_B T$ ) =  $\sum_{i,j} f_R(i) f_S(i,j) f_T(j)$
- Sketch-based solution
  - Compute random variables  $X_R$ ,  $X_S$  and  $X_T$

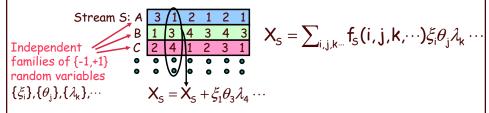


# AMS Sketching for Multi-Join Aggregates





- Sketches can be used to compute answers for general multi-join COUNT queries (over streams R, S, T, ......)
  - For each pair of attributes in equality join constraint, use independent family of {-1, +1} random variables
  - Compute random variables  $X_R, X_S, X_T, .....$



(E[X]= COUNT(R⋈S⋈T⋈ ......)) - Return X=X<sub>R</sub>X<sub>S</sub>X<sub>T</sub> ......  $Var[X] \le 2^{2m} \cdot SJ(R) \cdot SJ(S) \cdot SJ(T) \cdots$ 

- Explosive increase with the number of joins!

# Boosting Accuracy by Sketch Partitioning: Basic Idea

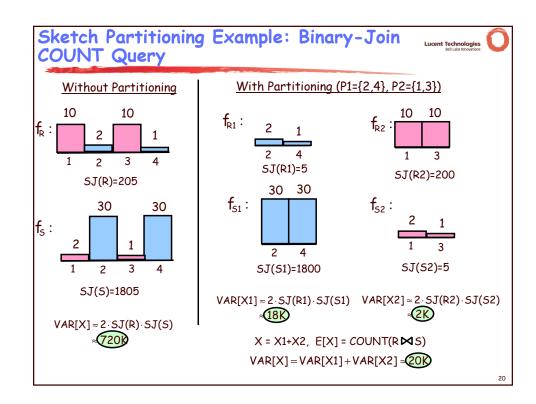


• For  $\varepsilon$  error, need  $Var[Y] \le \frac{\varepsilon^2 COUNT^2}{8}$ 

$$S = \frac{8 \cdot (2^{2m} SJ(R) \cdot SJ(S) \cdots)}{\epsilon^2 COUNT^2}$$
 copies

$$Var[Y] = \frac{Var[X]}{s} \le \frac{\epsilon^2 COUNT^2}{8}$$

- <u>Key Observation</u>: Product of self-join sizes for <u>partitions</u> of streams can be *much smaller* than product of self-join sizes for streams
  - Reduce space requirements by partitioning join attribute domains
    - · Overall join size = sum of join size estimates for partitions
  - Exploit coarse statistics (e.g., histograms) based on historical data or collected in an initial pass, to compute the best partitioning



# Overview of Sketch Partitioning



- Maintain independent sketches for partitions of join-attribute space
- Improved error guarantees
  - $Var[X] = \sum Var[Xi]$  is smaller (by intelligent domain partitioning)
  - "Variance-aware" boosting: More space to higher-variance partitions
- <u>Problem:</u> Given total sketching space S, find domain partitions p1,..., pk and space allotments s1,...,sk such that  $\sum_j sj \leq S$ , and the variance

$$\frac{\text{Var}[X1]}{s1} + \frac{\text{Var}[X2]}{s2} + \dots + \frac{\text{Var}[Xk]}{sk}$$
 is minimized

- Solved optimal for binary-join case (using Dynamic-Programming)
- -NP-hard for  $\geq 2$  joins
  - Extension of our DP algorithm is an effective heuristic -- optimal for independent join attributes
- Significant accuracy benefits for small number (2-4) of partitions

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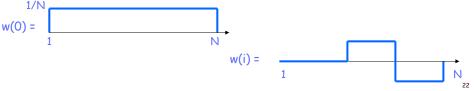
# Other Applications of AMS Stream Sketching



- Key Observation:  $|R1 \bowtie R2| = \sum f_1(i) f_2(i) = \langle f_1, f_2 \rangle = inner \ product!$
- $\bullet$  General result: Streaming  $(\varepsilon,\delta)$  estimation of "large" inner products using AMS sketching
- Other streaming inner products of interest
  - Top-k frequencies [CCF02]
    - Item frequency = < f, "unit\_pulse" >



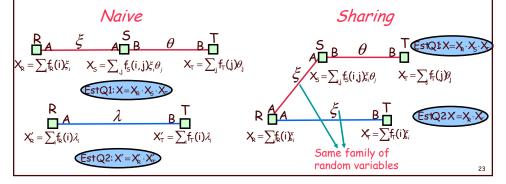
- Large wavelet coefficients [GKMS01]
  - Coeff(i) =  $\langle f, w(i) \rangle$ , where w(i) = i-th wavelet basis vector



#### More Recent Results on Stream Joins



- Better accuracy using "skimmed sketches" [GGR04]
  - "Skim" dense items (i.e., large frequencies) from the AMS sketches
  - Use the "skimmed" sketch only for sparse element representation
  - Stronger worst-case guarantees, and much better in practice
    - · Same effect as sketch partitioning with no apriori knowledge!
- Sharing sketch space/computation among multiple queries [DGGR04]



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#### Distinct Value Estimation



- <u>Problem:</u> Find the *number of distinct values* in a stream of values with domain [0,...,N-1]
  - Zeroth frequency moment  $F_{\scriptscriptstyle 0}$  , LO (Hamming) stream norm
  - Statistics: number of species or classes in a population
  - Important for query optimizers
  - Network monitoring: distinct destination IP addresses, source/destination pairs, requested URLs, etc.
- Example (N=64) Data stream: 3 0 5 3 0 1 7 5 1 0 3 7

Number of distinct values: 5

- Hard problem for random sampling! [CCMN00]
  - Must sample almost the entire table to guarantee the estimate is within a factor of 10 with probability > 1/2, regardless of the estimator used!

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# Hash (aka FM) Sketches for Distinct Value Estimation [FM85]



- Assume a hash function h(x) that maps incoming values x in [0,..., N-1] uniformly across [0,..., 2^L-1], where L = O(logN)
- Let lsb(y) denote the position of the least-significant 1 bit in the binary representation of y
  - A value x is mapped to lsb(h(x))
- Maintain Hash Sketch = BITMAP array of L bits, initialized to 0
  - For each incoming value x, set BITMAP[ lsb(h(x)) ] = 1

 $x = 5 \longrightarrow h(x) = 101100 \longrightarrow lsb(h(x)) = 2$ 

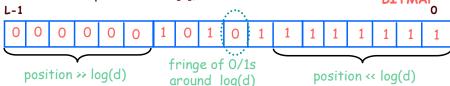
 5
 4
 3
 2
 1
 0

 0
 0
 0
 1
 0
 0

# Hash (aka FM) Sketches for Distinct Value Estimation [FM85]



- By uniformity through h(x): Prob[BITMAP[k]=1] = Prob[ $10^k$ ] =  $\frac{1}{2^{k+1}}$ 
  - Assuming d distinct values: expect d/2 to map to BITMAP[0], d/4 to map to BITMAP[1], ...



- Let R = position of rightmost zero in BITMAP
  - Use as indicator of log(d)
- [FM85] prove that E[R] =  $\log(\phi d)$  , where  $\phi = .7735$ 
  - Estimate  $d = 2^R/\phi$
  - Average several iid instances (different hash functions) to reduce estimator variance

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# Hash Sketches for Distinct Value Estimation

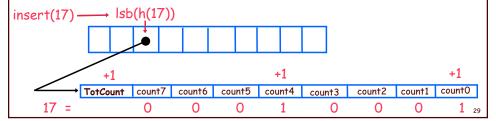


- [FM85] assume "ideal" hash functions h(x) (N-wise independence)
- [AMS96]: pairwise independence is sufficient
  - h(x) =  $(a \cdot x + b) \mod N$  , where a, b are random binary vectors in [0,...,2^L-1]
  - Small-space  $(\mathcal{E},\delta)$  estimates for distinct values proposed based on FM ideas
- Delete-Proof: Just use counters instead of bits in the sketch locations
  - +1 for inserts, -1 for deletes
- Composable: Component-wise OR/add distributed sketches together
  - Estimate  $|S1 \cup S2 \cup ... \cup Sk| = set$ -union cardinality

## Processing Set Expressions over Update Streams [GGR03]



- Estimate cardinality of *general set expressions* over streams of updates
  - E.g., number of distinct (source,dest) pairs seen at both R1 and R2 but not R3? | (R1 ∩ R2) - R3 |
- 2-Level Hash-Sketch (2LHS) stream synopsis: Generalizes FM sketch
  - First level:  $\Theta(\log N)$  buckets with exponentially-decreasing probabilities (using lsb(h(x)), as in FM)
  - Second level: Count-signature array (logN+1 counters)
    - · One "total count" for elements in first-level bucket
    - · logN "bit-location counts" for 1-bits of incoming elements



#### Processing Set Expressions over Update Streams: Key Ideas





• Build several independent 2LHS, fix a level I, and look for singleton first-level buckets at that level I



 Singleton buckets and singleton element (in the bucket) are easily identified using the count signature

Singleton bucket count signature 

- Singletons discovered form a distinct-value sample from the union of the streams
  - Frequency-independent, each value sampled with probability  $\frac{1}{2^{l+1}}$
- Determine the fraction of "witnesses" for the set expression E in the sample, and scale-up to find the estimate for |E|

# Example: Set Difference, |A-B|



- Parallel (same hash function), independent 2LHS synopses for input streams A, B
- Assume robust estimate  $\hat{u}$  for  $|A \cup B|$  (using known FM techniques)
- Look for buckets that are *singletons for*  $A \cup B$  at level  $| = | \log \hat{u} |$ 
  - Prob[singleton at level 1] > constant (e.g., 1/4)
  - Number of singletons (i.e., size of distinct sample) is at least a constant fraction (e.g., > 1/6) of the number of 2LHS (w.h.p.)
- "Witness" for set difference A-B: Bucket is singleton for stream A and empty for stream B
  - -Prob[witness | singleton] =  $|A-B| / |A \cup B|$
- Estimate for  $|A-B| = \frac{\# \text{ witnesses for } A-B}{\# \text{ singleton buckets}} \times \hat{u}$

# Estimation Guarantees





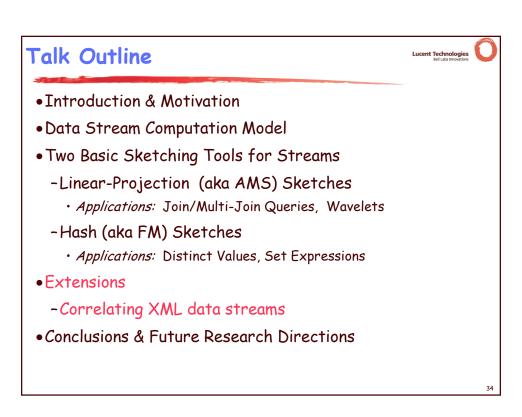
- ullet Our set-difference cardinality estimate is within a relative error of ulletwith probability  $\geq 1-\delta$  when the number of 2LHS is  $O(\frac{|A \cup B| \log(1/\delta)}{|A-B|\epsilon^2})$
- Lower bound of  $\Omega(\frac{|A \cup B|}{|A B|\epsilon})$  space, using communication-complexity arguments
- Natural generalization to arbitrary set expressions E = f(S1,...,Sn)
  - Build parallel, independent 2LHS for each S1,..., Sn
  - Generalize "witness" condition (inductively) based on E's structure
  - $(\varepsilon, \delta)$  estimate for |E| using  $O(\frac{|S1 \cup ... \cup Sn| \log(1/\delta)}{|F| \varepsilon^2})$ 2LHS synopses
- Worst-case bounds! Performance in practice is much better [GGR03]

#### Application: Detecting TCP-SYN-Flooding Lucent Technologies DDoS Attacks · Monitor potential DDoS activity over large ISP network - cannot maintain state for each potential destination/victim • Top-k based on traffic volume gives high traffic destinations (e.g., Yahoo!) - Attack traffic may not be high - Cannot distinguish attacks from "flash crowds" • "Right" metric: Top-k destinations wrt number of distinct connecting sources - Deletions to remove legitimate TCP connections from synopses Novel, space/time efficient, hash-based streaming algorithm - 2LHS used as a component for distinct-value estimation DDoS Spoofed IP sources Monitor Flow update Attack Mechanism streams • Flood of small SYN packets to victim from spoofed source addrs SYN-ACK responses to spoofed

IP sources

Resources exhausted

Many "half-open" connections



Victim

# Processing XML Data Streams

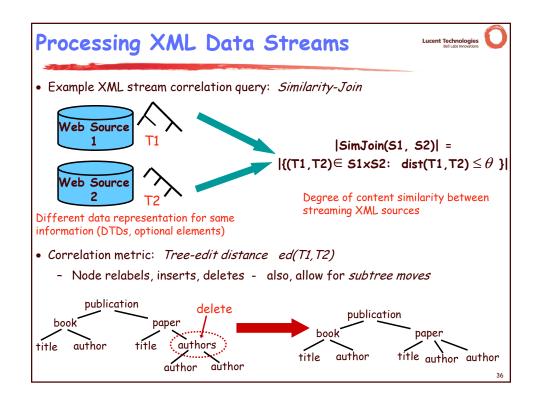


- XML: Much richer, (semi)structured data model
  - Ordered, node-labeled data trees
- Bulk of work on XML streaming: Content-based filtering of XML documents (publish/subscribe systems)
  - Quickly match incoming documents against standing XPath subscriptions



(X/Yfilter, Xtrie, etc.)

- Essentially, simple selection queries over a stream of XML records!
- No work on more complex XML stream queries
  - For example, queries trying to correlate different XML data streams

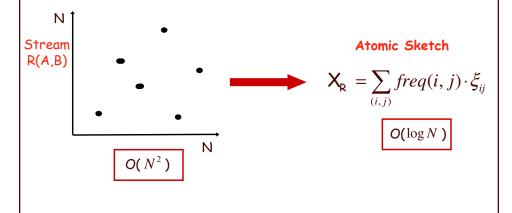


# How About AMS Sketches?





- Randomized linear projections (aka AMS sketches) are useful for points over a numeric vector space
  - Not structured objects over a complex metric space (tree-edit distance)

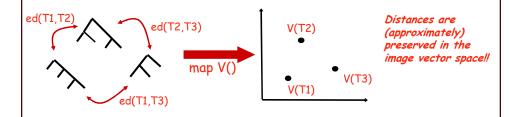


# Our Approach [GK03]





• Key idea: Build a low-distortion embedding of streaming XML and the tree-edit distance metric in a multi-d normed vector space



- Given such an embedding, sketching techniques now become relevant in the streaming XML context!
  - E.g., use AMS sketches to produce synopses of the data distribution in the image vector space

# Our Approach [GK03] (cont.)



- Construct low-distortion embedding for tree-edit distance over streaming XML documents -- Requirements:
  - Small space/time
  - Oblivious: Can compute V(T) independent of other trees in the stream(s)
  - Bourgain's Lemma is inapplicable!
- First algorithm for low-distortion, oblivious embedding of the tree-edit distance metric in small space/time
  - Fully deterministic, embed into L1 vector space
  - Bound of  $O(\log^2 n \log^* n)$  on distance distortion for trees with  $\leq$  n nodes

$$\|V(S) - V(T)\|_{1} = \sum |V(S)[i] - V(T)[i]| = O(\log^{2} n \log^{*} n) \cdot ed(S, T)$$

- Worst-case bound! Distortions much smaller over real-life data
  - Factors of 5-10 for 15K-node trees, consistently overestimate

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# Our Approach [GK03] (cont.)



- Applications in XML stream query processing
  - Combine our embedding with existing pseudo-random linearprojection sketching techniques
  - Build a small-space sketch synopsis for a massive, streaming XML data tree
    - Concise surrogate for tree-edit distance computations
  - Approximating tree-edit distance similarity joins over XML streams in small space/time
- First algorithmic results on correlating XML data in the streaming model
- Other important algorithmic applications for our embedding result
  - Approximate tree-edit distance in (near-linear)  $O(n\log^* n)$  time

# Embedding Algorithm



- Key Idea: Given an XML tree T, build a hierarchical parsing structure over T by intelligently grouping nodes and contracting edges in T
  - At parsing level i: T(i) is generated by grouping nodes of T(i-1) (T(0) = T)
  - Each node in the parsing structure (T(i), for all i = 0, 1, ...) corresponds to a connected subtree of T
  - Vector image V(T) is basically the *characteristic vector* of the resulting multiset of subtrees (in the entire parsing structure)

V(T)[x] = no. of times subtree x appears in the parsing structure for T

- Our parsing guarantees
  - O(log|T|) parsing levels (constant-fraction reduction at each level)
  - V(T) is very sparse: Only O(|T|) non-zero components in V(T)
    - Even though dimensionality =  $O((4|\sigma|)^n)$  ( $\sigma$  = label alphabet)
    - · Allows for effective sketching
  - V(T) is constructed in time  $O(|T| \log^* |T|)$

# Embedding Algorithm (cont.)

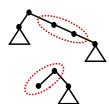




- Node grouping at a given parsing level T(i): Create groups of 2 or 3 nodes of T(i) and merge them into a single node of T(i+1)
  - 1. Group maximal sequence of contiguous leaf children of a node



- 2. Group maximal sequence of contiguous nodes in a chain



- 3. Fold leftmost lone leaf child into parent
- Grouping for Cases 1,2: Deterministic coin-tossing process of Cormode and Muthukrishnan [SODA'02]
  - Key property: Insertion/deletion in a sequence of length k only affects the grouping of nodes in a radius of  $\log^* k + 5$  from the point of change

# • Example hierarchical tree parsing $T(0) = T \qquad T(1) \qquad T(2) \qquad T(3)$ • V(T)[y] += 1• V(T)[x] += 1

# Main Embedding Result





• Theorem: Our embedding algorithm builds a vector V(T) with O(|T|) non-zero components in time  $O(|T|\log^*|T|)$ ; further, given trees T, S with n = max{|T|, |S|}, we have:

$$ed(S,T) \le 5 \cdot ||V(S) - V(T)||_1 = O(\log^2 n \log^* n) \cdot ed(S,T)$$

- Upper-bound proof highlights
  - Key Idea: Bound the size of "influence region" (i.e., set of affected node groups) for a tree-edit operation on T (=T(0)) at each level of parsing
    - We show that this set is of size  $O(i \log^* n)$  at level i
  - Then, it is simple to show that any tree-edit operation can change  $\|V(T)\|_1$  by at most  $O(\log^2 n \log^* n)$ 
    - L1 norm of subvector at level i changes by at most O(|influence region|)

# Main Embedding Result (cont.)



- · Lower-bound proof highlights
  - Constructive: "Budget" of at most  $5 \cdot \|V(S) V(T)\|_1$  tree-edit operations is sufficient to convert the parsing structure for S into that for T
    - · Proceed bottom up, level-by-level
    - At bottom level (T(0)), use budget to insert/delete appropriate labeled nodes
    - · At higher levels, use subtree moves to appropriately arrange nodes
- See PODS'03 paper for full details . . .

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# Sketching a Massive, Streaming XML Tree use the late transferred to the control of the control o

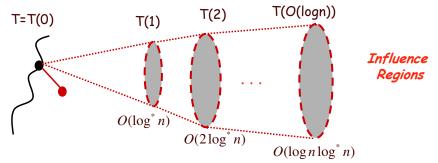


- Input: Massive XML data tree  $T (n = |T| \Rightarrow available memory)$ , seen in preorder (e.g., SAX parser output)
- Output: Small space surrogate (vector) for high-probability, approximate tree-edit distance computations (to within our distortion bounds)
- Theorem: Can build a  $O(\log \frac{1}{\delta})$ -size sketch vector of V(T) for approximate tree-edit distance computations in  $O(d \log^2 n (\log^* n)^2)$  space and  $O(\log d \log^2 n (\log^* n)^2)$  time per element
  - d = depth of T,  $\delta$  = probabilistic confidence in ed() approximation
  - XML trees are typically "bushy" (d<<n or d = O(polylog(n)))</li>

#### Sketching a Massive, Streaming XML Tree (cont.)



- Key Ideas
  - Incrementally parse T to produce V(T) as elements stream in
  - Just need to retain the *influence region* nodes for each parsing level and for each node in the current root-to-leaf path



- While updating V(T), also produce an L1 sketch of the V(T) vector using the techniques of Indyk [FOCS'00]

#### Approximate Similarity Joins over XML Streams





|SimJoin(S1, S2)| =

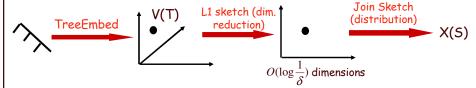
 $|\{(T1,T2) \in S1xS2: ed(T1,T2) \leq \theta \}|$ 

- Input: Long streams S1, S2 of N (short) XML documents ( $\leq$  b nodes)
- Output: Estimate for |SimJoin(S1, S2)|
- Theorem: Can build an atomic sketch-based estimate for SimJoin(S1, S2) where distances are approximated to within  $O(\log^2 b \log^* b)$  in space  $O(b + \log \frac{1}{\delta} \log N)$  and  $O(\frac{1}{\delta} + b \log^* b)$ time per document
  - $\delta$  = probabilistic confidence in distance estimates

#### Approximate Similarity Joins over XML Streams (cont.)



- Key Ideas
  - Our embedding of streaming document trees, plus two distinct levels of sketching
    - · One to reduce L1 dimensionality, one to capture the data distribution (for joining)
    - · Finally, similarity join in lower-dimensional L1 space



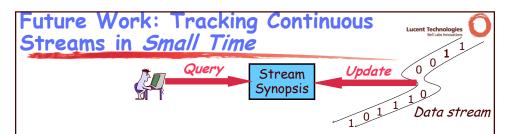
- · Some technical issues: high-probability L1 dimensionality reduction is not possible, sketching for L1 similarity joins
- Details in the paper . . .

# Conclusions



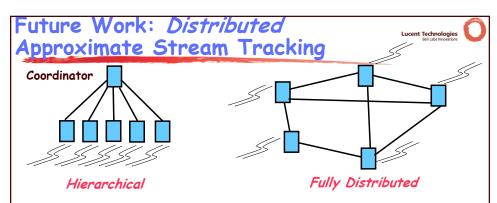


- Analyzing massive data streams: Real problem with several real-world applications
  - Fundamentally rethink data management under stringent constraints
  - Single-pass algorithms with limited memory/CPU-time resources
- Pseudo-random sketching is a viable technique for a variety of streaming tasks
  - Limited-space randomized approximations
  - Probabilistic guarantees on the quality of the approximate answer
  - Delete-proof (supports insertion and deletion of records)
  - Composable (ideal for distributed computation)



- Update/Query times are typically  $\Omega(|synopsis|)$  -- fine as long as synopsis sizes are small (polylog), BUT...
  - Small synopses are often *impossible* (strong communication-complexity lower bounds)
    - E.g., set expressions, joins, . . .
- Synopsis size may not be the crucial limiting factor (PCs with Gigabytes of RAM)
- Guaranteed small (polylog) update/query times are critical for high-speed streams
  - Time-efficient streaming algorithms --  $\Omega(|\text{synopsis}|)$  times are not adequate!
  - Have some initial results for small-time tracking of set expressions and joins

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- Goal: Effective tracking of a global quantity/query over the union of a distributed collection of streams
  - Composability of sketches makes them ideal for distributed computation
  - Additional concern: Communication Efficiency
    - · Minimize message exchanges involved for a given accuracy guarantee
- Some initial results on distributed "top-k frequency" monitoring [BO03]
  - Deterministic quarantees, using full space -- no sketching/synopses employed
  - More complex distributed tracking problems (e.g., joins) are wide open!

# Other Future Research Directions



- Sketches/synopses for richer types of streaming data and queries
  - Spatial data streams, queries over sliding windows, mining/querying streaming graphs, . . .
- Other metric-space embeddings in the streaming model
- Stream-data processing architectures and query languages
  - Progress: Aurora, STREAM, Telegraph, . . .
- Integration of streams and static relations
  - Effect on DBMS components (e.g., query optimizer)
- Novel, important application domains
  - Sensor networks, financial analysis, security, . . .

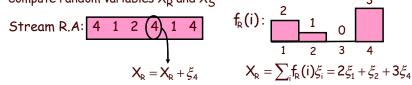
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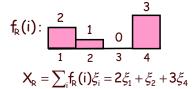
# Thank you! http://www.bell-labs.com/~minos/ minos@research.bell-labs.com

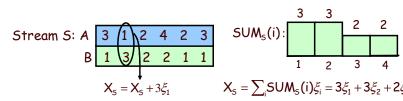
## Using Sketches to Answer SUM Queries

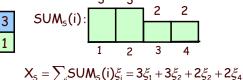


- <u>Problem:</u> Compute answer for query  $SUM_B(R \bowtie_A S) = \sum_i f_R(i) \cdot SUM_S(i)$ 
  - $SUM_S(i)$  is sum of B attribute values for records in S for whom S.A = i
- Sketch-based solution
  - Compute random variables  $X_R$  and  $X_S$









- Return  $X=X_{R}X_{S}$  $(E[X] = SUM_R(R \bowtie_A S))$ 

#### Stream Wavelet Approximation using AMS Sketches [GKMS01]



- Single-join approximation with sketches [AGMS99]
  - Construct approximation to  $|R1 \bowtie R2| = \sum f_1(i)f_2(i)$  within a relative error of  $\mathcal{E}$  with probability  $\geq 1-\delta$  using space  $O(\log N \cdot \log(1/\delta) / (\varepsilon^2 \lambda^2))$  , where

$$\lambda \leq \frac{|\sum f_1(i)f_2(i)|}{\sqrt{\sum f_1^2(i) \cdot \sum f_2^2(i)}} = |\mathsf{R1} \bowtie \mathsf{R2}| / \mathsf{Sqrt}(\prod \mathsf{self-join \, sizes})$$

- Observation:  $|R1 \bowtie R2| = \sum f_1(i) f_2(i) = \langle f_1, f_2 \rangle = inner product!!$ 
  - General result for inner-product approximation using sketches
- Other inner products of interest: Haar wavelet coefficients!
  - Haar wavelet decomposition = inner products of signal/distribution with specialized (wavelet basis) vectors

## Space Allocation Among Partitions



• <u>Key Idea</u>: Allocate more space to sketches for partitions with higher variance

$$Var[Y] = \frac{Var[X1]}{s1} + \frac{Var[X2]}{s2} \le \frac{\epsilon^2 COUNT^2}{8}$$

- Example: Var[X1]=20K, Var[X2]=2K
  - For s1=s2=20K, Var[Y] = 1.0 + 0.1 = 1.1
  - For s1=25K, s2=8K, Var[Y] = 0.8 + 0.25 = 1.05

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## Sketch Partitioning Problems



• <u>Problem 1:</u> Given sketches X1, ...., Xk for partitions P1, ..., Pk of the join attribute domain, what is the space sj that must be allocated to Pj (for sj copies of Xj) so that

$$\begin{aligned} & \text{Var}[Y] = \frac{\text{Var}[X1]}{s1} + \frac{\text{Var}[X2]}{s2} + \dots + \frac{\text{Var}[Xk]}{sk} \leq \frac{\epsilon^2 \ \text{COUNT}^2}{8} \\ & \text{and } \sum_i sj \ \text{is minimum} \end{aligned}$$

• <u>Problem 2</u>: Compute a partitioning P1, ..., Pk of the join attribute domain, and space sj allocated to each Pj (for sj copies of Xj) such that

$$\begin{aligned} & \text{Var[Y]} = \frac{\text{Var[X1]}}{\text{s1}} + \frac{\text{Var[X2]}}{\text{s2}} + \dots + \frac{\text{Var[Xk]}}{\text{sk}} \leq \frac{\epsilon^2 \ \text{COUNT}^2}{8} \\ & \text{and } \sum_{j} \text{sj} \quad \text{is minimum} \end{aligned}$$

• Solutions also apply to dual problem (Min. variance for fixed space)

## Optimal Space Allocation Among Partitions





• <u>Key Result (Problem 1):</u> Let X1, ...., Xk be sketches for partitions P1, ..., Pk of the join attribute domain. Then, allocating space

$$sj = \frac{8\sqrt{Var(Xj)}(\sum_{j}\sqrt{Var(Xj)})}{\epsilon^{2} COUNT^{2}}$$

to each Pj (for sj copies of Xj) ensures that  $Var[Y] \leq \frac{\epsilon^2 \ COUNT^2}{8}$  and  $\sum_j sj$  is minimum

- Total sketch space required:  $\sum_{j} sj = \frac{8(\sum_{j} \sqrt{Var(Xj)})^2}{\epsilon^2 COUNT^2}$
- $\begin{array}{l} \bullet \, \underline{\text{Problem 2 (Restated):}} \,\, \text{Compute a partitioning P1, ..., Pk of the join attribute domain such that} \, \sum_{i} \sqrt{\text{Var}(Xj)} \,\, \text{is minimum} \end{array}$ 
  - Optimal partitioning P1, ..., Pk minimizes total sketch space

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# Binary-Join Queries: Binary Space Partitioning

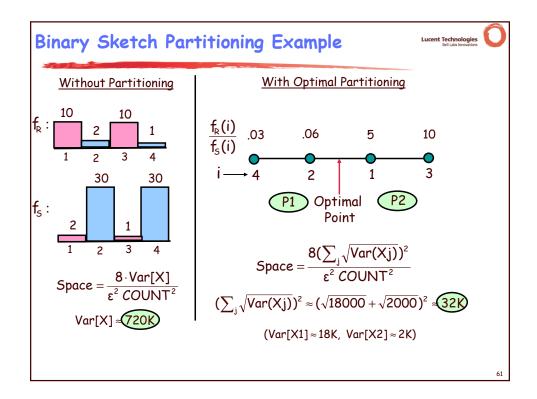


- <u>Problem:</u> For COUNT(R  $\bowtie_A$  S), compute a partitioning P1, P2 of A's domain {1, 2, ..., N} such that  $\sqrt{\text{Var}(X1)} + \sqrt{\text{Var}(X2)}$  is minimum
  - Note:  $Var(Xj) \approx 2\sum_{i \in P_j} f_R(i)^2 \sum_{i \in P_j} f_S(i)^2$
- Key Result (due to Breiman): For an optimal partitioning P1, P2,

$$\forall i1 {\in} \ P1, \ \forall i2 {\in} \ P2, \quad \frac{f_R(i1)}{f_S(i1)} {<} \frac{f_R(i2)}{f_S(i2)}$$

- Algorithm
  - Sort values i in A's domain in increasing value of  $\frac{f_R(i)}{f_L(i)}$
  - Choose partitioning point that minimizes

$$\sqrt{2 {\sum\nolimits_{i \in P1} {{f_R}(i)^2} {\sum\nolimits_{i \in P1} {{f_S}(i)^2} } } } + \sqrt{2 {\sum\nolimits_{i \in P2} {{f_R}(i)^2} } {\sum\nolimits_{i \in P2} {{f_S}(i)^2} }$$







- <u>Problem:</u> For COUNT(R $\bowtie_A$ S), compute a partitioning P1, P2, ..., Pk of A's domain such that  $\sum_j \sqrt{\text{Var}(Xj)}$  is minimum
- Previous result (for 2 partitions) generalizes to k partitions
- Optimal k partitions can be computed using Dynamic Programming
  - Sort values i in A's domain in increasing value of  $\frac{f_R(i)}{f_S(i)}$
  - Let  $\phi(u,t)$  be the value of  $\sum_j \sqrt{2 \sum_{i \in P_j} f_R(i)^2 \sum_{i \in P_j} f_S(i)^2}$  when [1,u] is split optimally into t partitions P1, P2, ...., Pt

$$\phi(u,t) = \min_{1 \le v \le u} \{\phi(v,t-1) + \sqrt{2 \sum_{i=v+1}^{u} f_{R}(i)^{2} \sum_{i=v+1}^{u} f_{R}(i)^{2}} \}$$

• Time complexity:  $O(kN^2)$ 

## Sketch Partitioning for Multi-Join Queries Lucent Technologies Bellular Invalidation





• <u>Problem:</u> For COUNT(R  $\bowtie_A$  S $\bowtie_B$ T), compute a partitioning  $P_1^A, P_2^A, ..., P_{k_a}^A(P_1^B, P_2^B, ..., P_{k_B}^B)$  of A(B)'s domain such that  $k_A k_B < k$ , and the following is minimum

$$\psi = \sum\nolimits_{P_{j}^{A}} \sum\nolimits_{P_{j}^{B}} \sqrt{\text{Var}(X_{(P_{i}^{A},P_{j}^{B})})}$$

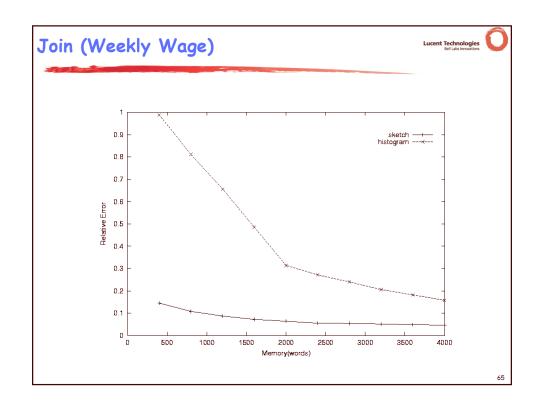
- Partitioning problem is NP-hard for more than 1 join attribute
- If join attributes are independent, then possible to compute optimal partitioning
  - Choose k1 such that allocating k1 partitions to attribute A and k/k1 to remaining attributes minimizes Ψ
  - Compute optimal k1 partitions for A using previous dynamic programming algorithm

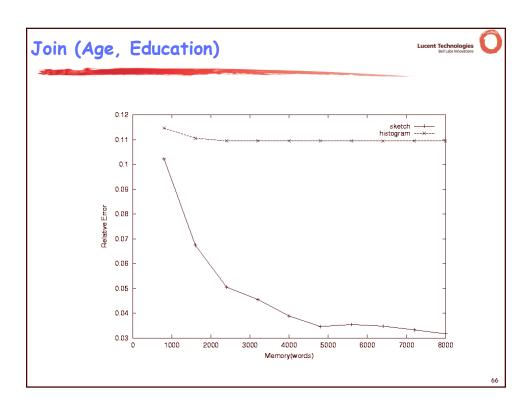
## **Experimental Study**

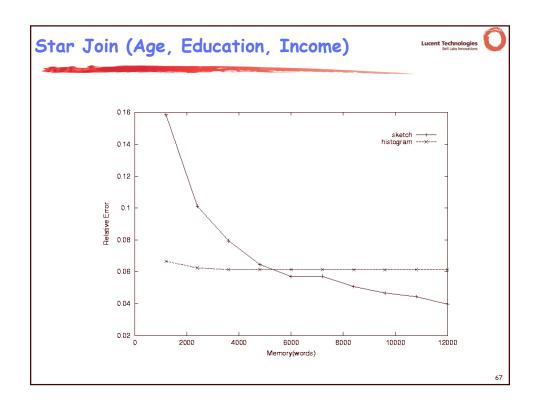


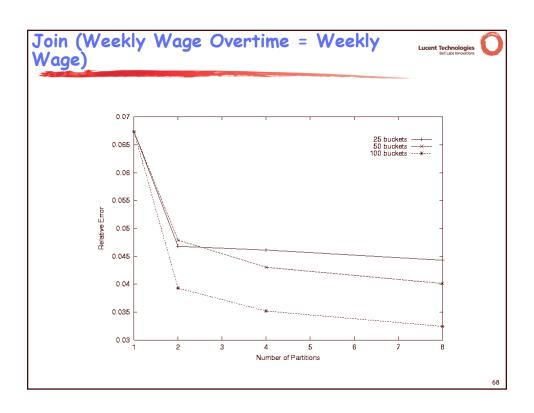


- Summary of findings
  - Sketches are superior to 1-d (equi-depth) histograms for answering COUNT queries over data streams
  - Sketch partitioning is effective for reducing error
- Real-life Census Population Survey data sets (1999 and 2001)
  - Attributes considered:
    - · Income (1:14)
    - · Education (1:46)
    - · Age (1:99)
    - · Weekly Wage and Weekly Wage Overtime (0:288416)
- Error metric: relative error ( | actual approx |









#### More work on Sketches...



- Low-distortion vector-space embeddings (JL Lemma) [Ind01] and applications
  - E.g., approximate nearest neighbors [IM98]
- Wavelet and histogram extraction over data streams [GGI02, GIM02, GKMS01, TGIK02]
- Discovering patterns and periodicities in time-series databases [IKM00, CIK021
- Quantile estimation over streams [GKMS02]
- Distinct value estimation over streams [CDI02]
- Maintaining top-k item frequencies over a stream [CCF02]
- Stream norm computation [FKS99, Ind00]
- Data cleaning [DJM02]

## Sketching for Multiple Standing Queries





- Consider queries Q1 = COUNT(R  $\bowtie_A$  S  $\bowtie_R$ T) and Q2 = COUNT(R  $\bowtie_{A=R}$ T)
- Naive approach: construct separate sketches for each join
  - $\xi$ ,  $\theta$ ,  $\lambda$  are independent families of pseudo-random variables

$$X_{R} = \sum_{i} f_{R}(i) \xi_{i} \qquad X_{S} = \sum_{i,j} f_{S}(i,j) \xi_{i} \theta_{j} \qquad X_{T} = \sum_{j} f_{T}(j) \theta_{j}$$

$$\text{Est Q1: } X = X_{R} \cdot X_{S} \cdot X_{T}$$

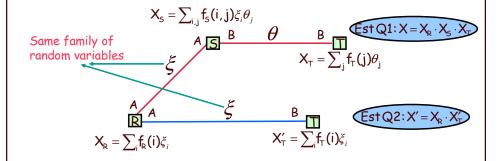
$$X'_{R} = \sum_{i} f_{R}(i) \lambda_{i}$$
 
$$X'_{T} = \sum_{i} f_{T}(i) \lambda_{i}$$
 
$$X'_{T} = \sum_{i} f_{T}(i) \lambda_{i}$$
 Est Q2:  $X' = X'_{R} \cdot X'_{T}$ 

# Sketch Sharing





- Key Idea: Share sketch for relation R between the two queries
  - Reduces space required to maintain sketches



- BUT, cannot also share the sketch for T!
  - Same family on the join edges of Q1

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# Sketching for Multiple Standing Queries





- Algorithms for sharing sketches and allocating space among the queries in the workload
  - Maximize sharing of sketch computations among queries
  - Minimize a cumulative error for the given synopsis space
- Novel, interesting combinatorial optimization problems
  - Several NP-hardness results :-)
- Designing effective heuristic solutions

#### Set Expressions to Sketch Expressions



- Given set expression E = f(S1,...,Sn), level of inference I
  - Again, look for buckets that are singletons for the union of S1,...,
     Sn at level I
- "Witness" Condition for E: Create boolean expression B(E) over parallel sketches inductively
  - 1. Replace Si by isSingleton(sketch(Si), I)
  - 2. Replace E1  $\cap$  E2 by B(E1) AND B(E2)
  - 3. Replace E1-E2 by B(E1) AND (NOT B(E2))
  - 4. Replace E1  $\bigcup$  E2 by B(E1) OR B(E2)
- Then, Prob[witness | singleton] = |E| / |S1∪ ... U Sn|

